

Spin alignment of vector mesons in heavy ion and proton - proton collisions

Alejandro Ayala^{1,2*}, Eleazar Cuautle¹, G. Herrera Corral^{3,4†}, J. Magnin² and Luis Manuel Montaño⁴

¹*Instituto de Ciencias Nucleares Universidad Nacional Autónoma de México,
Apartado Postal 70-543 México Distrito Federal 04510, México.*

²*Centro Brasileiro de Pesquisas Físicas CBPF,
Rua Dr. Xavier Sigaud 150 22290-180 Rio de Janeiro, Brazil.*

³*CERN, CH 1211 Geneva 23, Switzerland.*

⁴*Centro de Investigación y de Estudios Avanzados del IPN,
Apartado Postal 14-740 México Distrito Federal 07000, México.*

The spin alignment matrix element ρ_{00} for the vector mesons K^{*0} and $\phi(1020)$ has been measured in RHIC at central rapidities. These measurements are consistent with the absence of polarization with respect to the reaction plane in mid-central Au + Au collisions whereas, when measured with respect to the production plane in the same reactions and in p + p collisions, a non-vanishing and p_{\perp} -dependent ρ_{00} is found. We show that this behavior can be understood in a simple model of vector meson production where the spin of their constituent quarks is oriented during hadronization as the result of Thomas precession.

PACS numbers: 25.75.-q, 13.88.+e, 12.38.Mh, 25.75.Nq

The study of spin polarization of produced hadrons in reactions at high energies has opened a window to the understanding of the underlying dynamics of quark recombination. In the context of heavy-ion collisions, polarization studies can also help to understand the evolution of the system from its early stages [1, 2, 3].

Polarization analyses require to determine a given direction that serves as the spin quantization axis. From the experimental point of view, it is possible to determine two directions: the normal to the reaction and the normal to the production planes. The first plane is defined as the one containing the impact parameter and the beam direction vectors whereas the second one is defined as containing the hadron's final momentum and the beam direction vectors.

Recently, the STAR collaboration has reported measurements of the spin polarization properties of the vector mesons ϕ and K^* [4]. These measurements refer to the 00 component of the so called *spin alignment* density matrix ρ , which is the density matrix for a two-spin one-half system in a triplet state, expressed in terms of the coupled basis [5]. Recall that a value $\rho^{00} = 1/3$, means that the spin of the vector meson is not aligned with respect to the chosen quantization axis. Deviations from this value indicate a degree of polarization of the vector spin which ultimately might reflect a polarization of the constituent quarks.

The experimental findings reported can be summarized as follows: When the spin alignment is referred to the *reaction plane* in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV and measured at mid-rapidity, $\rho_{00} \simeq 1/3$, and it remains constant both as a function of p_{\perp} in the range $0 < p_{\perp} < 5$ GeV, for mid-central collisions, and as a function of the

average number of participants in the same p_{\perp} range, for both ϕ and K^* . When the spin alignment is referred to the *production plane* and measured at mid-rapidity, both for p + p and mid-central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, $\rho_{00} > 1/3$ and it has a concave shape as a function of p_{\perp} in the range $0 < p_{\perp} < 5$ GeV with minima at slightly different intermediate values of p_{\perp} for ϕ and K^* .

An interesting observation that can be inferred from the above listed results is that the dynamics of hadron formation seems to play a role to orient the spin of quarks that form meson.

A possible origin of a quark polarization driving the polarization of vector mesons is discussed in Refs. [1] which study the transfer of local relative angular momentum in peripheral nuclear collisions to the quark spin polarization by means of rescattering during the reaction. However, this mechanism also predicts a small global polarization growing almost linearly with impact parameter which, according to the aforementioned results is not observed in data.

Another interesting possibility that has only been studied in the context of hyperon polarization [6, 7] is the scenario where the spin of a quark is oriented during the recombination process. The semiclassical picture accounting for the polarization is the Thomas precession produced by the accelerating force that pulls a slow moving quark (q^s) to form a fast moving hadron [6]. This mechanism also predicts that if the quark is fast (q^f) and is decelerated to form the hadron, its polarization will be of opposite sign compared to the case when it is accelerated. The pulling force is required to not be parallel to the original quark velocity since the Thomas frequency is a vector formed by the cross product of the force \mathbf{F} and this velocity $\boldsymbol{\beta}$, namely

$$\boldsymbol{\omega}_T = \left(\frac{\gamma}{1 + \gamma} \right) \mathbf{F} \times \boldsymbol{\beta}, \quad (1)$$

*ayala@nucleares.unam.mx

†On leave of absence from CINVESTAV

where γ is the Lorentz gamma-factor. The polarization is given by [6]

$$\mathcal{P}^{s,f} = \mp \frac{\omega_T^{s,f}}{\Delta E}, \quad (2)$$

where the $- (+)$ sign refers to the q^s (q^f). $\omega_T^{s,f}$ is the magnitude of the Thomas precession frequency for q^s and q^f , respectively and ΔE is the change of energy in the process of hadron formation.

In this work we use the Thomas spin precession mechanism to describe the spin alignment of vector mesons produced at central rapidity in Au + Au and p + p collisions at $\sqrt{s_{NN}} = 200$. We show that under very simple assumptions, data for ρ_{00} are well reproduced within this approach.

The physical picture we use is that of a fast quark that decelerates and a slow one that accelerates to form a fast moving hadron. In the process, Thomas precession makes the spin of the former to acquire a positive polarization whereas the latter acquires a negative one. For central rapidities, the large momentum component of the hadron will thus be its transverse momentum p_{\perp}^H whereas the small component will be its longitudinal one, p_{\parallel}^H . We will assume that in the beam collision, a hard interaction produces a fast quark moving with a large transverse momentum p_{\perp}^f and, to simplify matters, a vanishing longitudinal momentum. This fast quark combines with the slow one, that we assume moves originally mainly in the longitudinal direction with momentum p_{\parallel}^s and, also for simplicity, take it with vanishing transverse momentum. This sharp difference in the original direction of motion of the recombining quarks is at the core of the produced polarization since, as we proceed to show, it gives rise to a distinct p_{\perp} dependence of ρ_{00} which seems to be also observed in data.

In order to form the hadron, which should move with an intermediate value of momentum, between that of the q^f and of the q^s , the fast quark should slow down whereas the slow quark should speed up. Notice that for this mechanism to work, there is no need to assume that the process happens only in either a proton-proton or a nucleus-nucleus collision. Notice also that the momentum of the formed hadron provides a fixed direction to define that q^s (q^f) decelerates (accelerates), whereas, when referred to the reaction plane, no such fixed direction exists, since the direction of the impact parameter vector changes from one reaction to another and in such situation either quark can accelerate or decelerate.

The pulling force is equal to the change in momentum $\Delta \mathbf{p}$ of the given quark, in the interval of time Δt for the recombination process to happen, that is

$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}. \quad (3)$$

Thus ω_T for the given quark can be computed as the

average over this time interval [6], namely

$$\begin{aligned} \omega_T^{s,f} &\propto \frac{\Delta p^{s,f}}{\Delta t} \beta^{s,f} \left(\int_{\Delta t} dt \sin \theta^{s,f} / \Delta t \right) \\ &\simeq \frac{\Delta p^{s,f}}{\Delta t} \beta^{s,f} \langle \sin \theta \rangle^{s,f}, \end{aligned} \quad (4)$$

where in the last line we have changed the time average of the sine of the angle by the average sine of the angle between the quark velocity vectors and their corresponding change in momentum. From Eqs. (2) and (4), we see that the calculation of the q^f and q^s polarizations reduces to computing the magnitudes of their change in momentum $\Delta p^{s,f}$, their $\langle \sin \theta \rangle^{s,f}$ and the change in energy ΔE .

We first compute the change in momenta. For q^s , we have

$$\Delta p^s = \sqrt{(p_{\parallel}^{s/H} - p_{\parallel}^s)^2 + (p_{\perp}^{s/H})^2}, \quad (5)$$

where $p_{\parallel, (\perp)}^{s/H}$ denote the parallel (transverse) component of the q^s in the hadron H . Let us assume that, in order to enhance the recombination probability, the rapidity of q^s has to be within the hadron's one. Under this assumption we can write

$$\begin{aligned} p_{\parallel}^s &= m_{\perp}^s \sinh y^H \\ &= \frac{m_{\perp}^s}{m_{\perp}^H} m_{\perp}^H \sinh y^H \\ &= \frac{m^s}{m_{\perp}^H} p_{\parallel}^H, \end{aligned} \quad (6)$$

where we have set $m_{\perp}^s = m^s$ since $p_{\perp}^s = 0$. Therefore, we can write

$$\begin{aligned} \Delta p^s &= \sqrt{[(x_{\parallel} - \frac{m^s}{m_{\perp}^H} p_{\parallel}^H)^2 + (x_{\perp} p_{\perp}^H)^2]} \\ &\simeq x_{\perp} p_{\perp}^H, \end{aligned} \quad (7)$$

where we have introduced the definitions for the momentum fractions that the q^s has inside the hadron,

$$\begin{aligned} x_{\parallel} &= p_{\parallel}^{s/H} / p_{\parallel}^H \\ x_{\perp} &= p_{\perp}^{s/H} / p_{\perp}^H, \end{aligned} \quad (8)$$

and have neglected the longitudinal hadron's momentum with respect to its transverse one.

Similarly, for the q^f change of momentum when decelerating, notice that we have

$$\Delta \mathbf{p}^f = \mathbf{p}_{\parallel}^{f/H} - \mathbf{p}_{\perp}^{f/H} - \mathbf{p}_{\perp}^f. \quad (9)$$

Since β^f initially points along the perpendicular direction and, although this velocity vector changes so that the final hadron's momentum eventually picks up a longitudinal component, the dominant component of the q^f

in the hadron is the transverse one. Therefore, for the cross product of $\Delta \mathbf{p}^f$ with β^f we get

$$\begin{aligned}\Delta \mathbf{p}^f \times \beta^f &= p_{\parallel}^{f/H} \beta^f \langle \sin \theta \rangle^f \\ &= (1 - x_{\parallel}) p_{\parallel}^H \langle \sin \theta \rangle^f,\end{aligned}\quad (10)$$

where we have enforced momentum conservation $x_{\perp} + x_{\parallel} = 1$ and have approximated $\beta^f \simeq 1$.

Now, we proceed to compute the average sine of the angles. To compute $\langle \sin \theta \rangle^s$, notice that given that initially β^s points along the parallel direction and $\Delta \mathbf{p}^s$ is basically directed along the perpendicular direction, the initial angle between these vectors is $\pi/2$ and the average one should be close to $\pi/4$. Thus $\langle \sin \theta \rangle^s \simeq 1/\sqrt{2}$. Rather than approximating β^s (and therefore γ^s), we introduce a factor a for this polarization and let this vary such that $0 < a < 1$. Thus we write

$$\begin{aligned}\omega^s &= a \frac{\Delta p^s}{\Delta t} \\ a &= \left(\frac{\gamma^s}{1 + \gamma^s} \right) \beta^s \langle \sin \theta \rangle^s.\end{aligned}\quad (11)$$

To compute $\langle \sin \theta \rangle^f$, notice that since $\Delta \mathbf{p}^f$ and β^f are almost perpendicular, $\langle \sin \theta \rangle^f \simeq 1$.

The change in energy is common to both the accelerating q^s and the decelerating q^f

$$\begin{aligned}\Delta E &= \{[(p_{\perp}^f)^2 + (p_{\parallel}^f)^2 + (m^f)^2]^{1/2} \\ &+ [(p_{\perp}^s)^2 + (p_{\parallel}^s)^2 + (m^s)^2]^{1/2} \\ &- [(p_{\perp}^H)^2 + (p_{\parallel}^H)^2 + (m^H)^2]^{1/2}\} \\ &\simeq \{[(p_{\perp}^f)^2 + (m^f)^2]^{1/2} + [(p_{\parallel}^s)^2 + (m^s)^2]^{1/2} \\ &- [(p_{\perp}^H)^2 + (m^H)^2]^{1/2}\} \\ &= \left\{ p_{\perp}^f \left[1 + \frac{(m^f)^2}{2(p_{\perp}^f)^2} \right] + [(p_{\parallel}^s)^2 + (m^s)^2]^{1/2} \right. \\ &\quad \left. - p_{\perp}^H \left[1 + \frac{(m^H)^2}{2(p_{\perp}^H)^2} \right] \right\},\end{aligned}\quad (12)$$

where we have set $p_{\perp}^s = p_{\parallel}^f = 0$, neglected $(p_{\parallel}^H)^2$ compared with $(p_{\perp}^H)^2$ and expanded the square roots assuming that the transverse momentum components are large. Introducing the relation between the hadron and q^f transverse momenta $p_{\perp}^f = p_{\perp}^H/z$, with $0 < z < 1$, we get

$$\begin{aligned}\Delta E &= \left\{ \frac{p_{\perp}^H}{z} \left(1 + \frac{z^2 (m^f)^2}{2(p_{\perp}^H)^2} \right) \right. \\ &\quad \left. + \left[\left(\frac{m^s}{m_{\perp}^H} \right)^2 (p_{\parallel}^H)^2 + (m^s)^2 \right]^{1/2} \right. \\ &\quad \left. - p_{\perp}^H \left(1 + \frac{(m^H)^2}{2(p_{\perp}^H)^2} \right) \right\}\end{aligned}\quad (13)$$

where we have made use of the assumption that the rapidity of the q^s coincides with that of the hadron. Notice

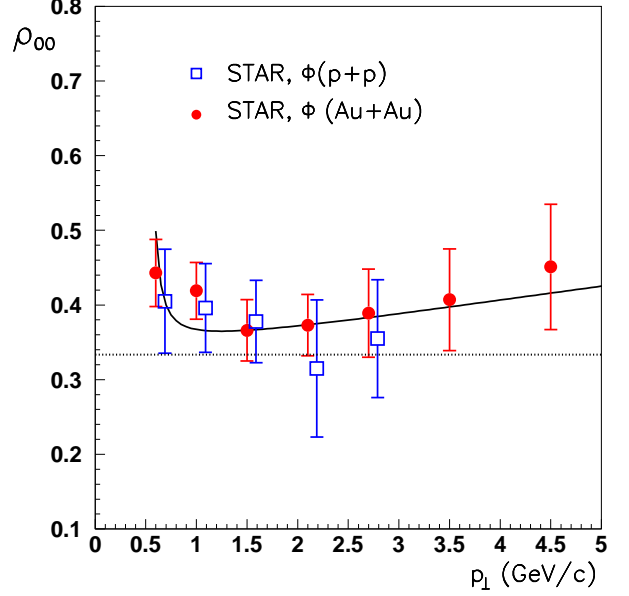


FIG. 1: (Color online) ρ_{00} as a function of p_{\perp} for $\phi(1020)$ using the model parameters described in the text, compared to data from STAR [4] for p + p and Au + Au collisions at centrality 20-60 %, measured with respect to the production plane. For clarity, the p_{\perp} for p + p data has been displaced by 0.09 GeV with respect to the reported central value. The statistical and systematic errors have been added in quadrature. For comparison, we also draw the constant value $1/3$ that represents the absence of polarization.

that the above expression can be simplified. In particular, since $p_{\parallel}^H = m_{\perp}^H \sinh y^H$, we can write

$$\left[\frac{(m^s)^2}{(m_{\perp}^H)^2} (p_{\parallel}^H)^2 + (m^s)^2 \right]^{1/2} = m^s \cosh y^H. \quad (14)$$

Therefore, ΔE can be expressed as

$$\begin{aligned}\Delta E &= \left\{ \left(\frac{1-z}{z} \right) p_{\perp}^H + \left[\frac{z(m^f)^2 - (m^H)^2}{2p_{\perp}^H} \right] \right. \\ &\quad \left. + m^s \cosh y^H \right\}.\end{aligned}\quad (15)$$

We emphasize that the approximation to compute ΔE is such that Eq. (15) is valid for $p_{\perp} \gtrsim m^H$ and that for $z \lesssim 1$ the validity of the approximation can be extended to lower values of p_{\perp} .

It is now easy to compute the polarization for the slow and fast quarks by means of Eq. (2) and from them, the ρ_{00} density matrix element given by

$$\rho_{00} = \frac{1 - \mathcal{P}^s \mathcal{P}^f}{3 + \mathcal{P}^s \mathcal{P}^f}. \quad (16)$$

Let us first proceed to apply the model to compute ρ_{00} for ϕ . This case is the simplest one to treat within our approach since the quark content of this particle is $s\bar{s}$ and

thus either one of these quarks can be thought of as being the fast (q^f) or the slow (q^s) one. Rather than making an exhaustive search in the parameter space, we choose reasonable values for them. We first fix the ϕ and strange quark masses to be $m^\phi = 1.02$ GeV, $m^{s,s} = 0.5$ GeV. The rapidity value we use is $y^H = 1$ and the formation time $\Delta t = 1$ fm. For the fractions of longitudinal and transverse momenta that the slow quark has inside the ϕ we take $x_\perp = x_\parallel = 0.5$. The fraction of the transverse momentum carried by the ϕ from the fast quark is taken as $z = 0.9$.

Figure 1 shows ρ_{00}^ϕ as a function of p_\perp^ϕ compared to data from STAR [4] for p + p and Au + Au collisions at centrality 20-60 %. A good description is obtained for $a = 0.25$.

We now proceed to apply this model to the case of K^* , whose quark content is $d\bar{s}$. However, in this case one needs to be careful since the symmetry between the masses, present in the description of ϕ , is absent. Consequently the spin alignment has to be treated in average. To this end, let us first take a simple scenario and consider the arithmetic average in the following manner

$$\rho_{00}^{K^*} = \frac{1}{2} \left(\rho_{00}^{f=s, s=d} + \rho_{00}^{f=d, s=s} \right). \quad (17)$$

Figure 2 shows $\rho_{00}^{K^*}$ as a function of $p_\perp^{K^*}$ compared to data from STAR [4] for p + p and Au + Au collisions at centrality 20-60%, measured in the production plane. The curves are computed such that we employ the same set of parameters as in the computation of ρ_{00}^ϕ (with $m^d = 0.3$ GeV) except for the value of z . The reason is that, whereas in the case that $q^f = s$ one can think that in order for this fast quark to pick up a d , the momenta of s and K^* are similar, when $q^f = d$, its momentum must be larger, given the mass difference between K^* and d . Thus if $q^f = s$ we choose $z = 0.9$ whereas when $q^f = d$ we use $z = 0.3$. The upper dashed curve represents the case for $\rho_{00}^{f=s, s=d}$, whereas the lower dashed curve is for $\rho_{00}^{f=d, s=s}$. The intermediate solid curve represents $\rho_{00}^{K^*}$ as the algebraic average of the above, as in Eq. (17).

An alternative approach is to consider that the $\rho_{00}^{K^*}$ can be computed by the substitution

$$\mathcal{P}^s \mathcal{P}^f \rightarrow \frac{\mathcal{P}^{s=s} \mathcal{P}^{f=d} + \mathcal{P}^{s=d} \mathcal{P}^{f=s}}{2}, \quad (18)$$

that is, by the *average product of polarizations*, in Eq. (16). Figure 2 shows also this possibility represented by the intermediate dotted curve, using the same set of parameters for the cases where $q^f = s$, $q^s = d$ and $q^f = d$, $q^s = s$, as discussed above. As can be seen from the figure, no significant difference is found with the case where the average is taken with the functions ρ_{00} and both approaches give a good description of data.

In conclusion, we have shown that data on the vector mesons ϕ and K^* spin alignment with respect to the production plane in Au + Au and p + p collisions are well

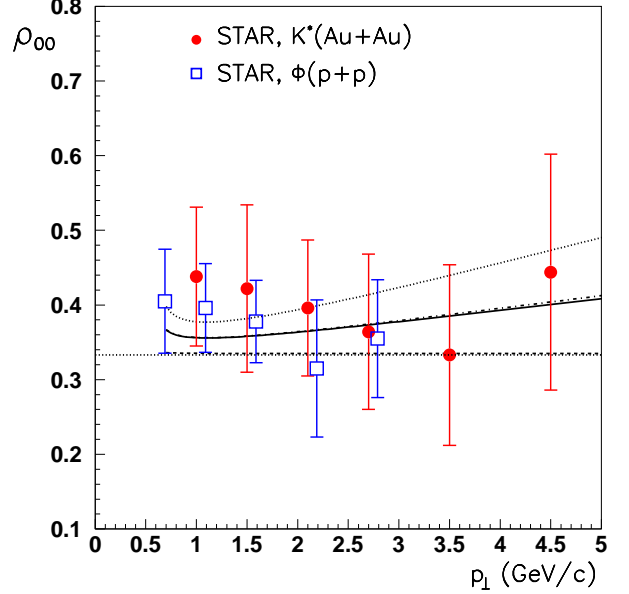


FIG. 2: (Color online) ρ_{00} as a function of p_\perp for K^* using the model parameters described in the text, compared to data from STAR [4] for p + p and Au + Au collisions at centrality 20-60 %, measured with respect to the production plane. For clarity, the p_\perp for p + p data has been displaced by 0.09 GeV with respect to the reported central value. The statistical and systematic errors have been added in quadrature. The upper dashed curve represents the case for $\rho_{00}^{f=s, s=d}$, whereas the lower dashed curve represents the case for $\rho_{00}^{f=d, s=s}$. The intermediate solid curve represents $\rho_{00}^{K^*}$ as the algebraic average of the above. The intermediate dotted curve represents the case where ρ_{00} is computed using the average product of polarizations. For comparison, we also draw the constant value $1/3$ that represents the absence of polarization.

described by assuming that these hadrons are produced by the recombination of a slow and a fast quark that in the process become polarized in opposite directions due to Thomas precession. In this plane, the momentum of the hadron provides a fixed direction to define whether a valence quark accelerates or decelerates. The mechanisms also clarifies the fact that when the spin alignment is referred to the reaction plane, ρ_{00} vanishes, given that no such fixed direction exists, since impact parameter vector changes from one reaction to another.

In the near future ALICE at the LHC will have the capability to measure and reconstruct ϕ and K^* mesons with larger statistics [8]. In addition, its particle identification will allow these meson's p_\perp to be measured beyond 5 GeV, well into the region where energy losses become important and also where fragmentation (as opposed to the recombination picture we are using here), becomes the dominant particle production mechanism. It will thus be interesting to study how the polarization of vector mesons changes when including these effects.

This is work for the future.

Acknowledgements

A.A. thanks the kind hospitality of faculty and staff members at CBPF during a sabatical visit. Support

for this work has been received in part by FAPERJ under Proj. No. E-26/110.166/2009, CNPq, the Brazilian Council for Science and Technology, PAPIIT-UNAM under grant Nos. IN116008 and IN116508 and by CONACyT-México.

-
- [1] A.-T. Liang and X.-N. Wang, Phys. Rev. Lett. **94**, 102301 (2005); Phys. Lett. B **629**, 20, (2005).
 - [2] B. Betz, M. Gyulassy, and G. Torrieri, Phys. Rev. C **76**, 044901 (2007).
 - [3] F. Becattini, F. Piccinini, and J. Rizzo, Phys. Rev. C **77**, 024906 (2008).
 - [4] B. I. Abelev *et al.* (STAR Collaboration), Phys. Rev. C **77**, 061902 (2008); J.H. Chen *et al.* (STAR Collaboration), J. Phys. G: Nucl. Phys. **35**, 044068 (2008); J.H. Chen *et al.* (STAR Collaboration), J. Phys. G: Nucl. Phys. **34**, S331 (2007).
 - [5] K. Schilling, P. Seyboth, and G. Wolf, Nucl. Phys. B **15**, 397 (1970); Erratum: Nucl. Phys. B **18**, 332 (1970).
 - [6] T. A. DeGrand and H. I. Miettinen, Phys. Rev. D **24**, 2419 (1981).
 - [7] A. Ayala, E. Cuautle, G. Herrera and L. M. Montaño, Phys. Rev. C **65**, 024902 (2002).
 - [8] ALICE: Physics Performance Report Vol. II J. Phys. G: Nucl. Part. Phys. **32**, 1295 (2006).